

# REPORT No. 461

## INTERFERENCE ON AN AIRFOIL OF FINITE SPAN IN AN OPEN RECTANGULAR WIND TUNNEL

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### SUMMARY

*The wall interference on an airfoil of finite span in an open-throat rectangular section has been treated theoretically and the result is presented in a convenient formula. Numerical results are given in tables and diagrams.*

### INTRODUCTION

Recently a number of investigators have been engaged in the study of wind-tunnel wall interference. Until a short time ago the only results available were the interference in a circular section by Prandtl (reference 1) and the closed rectangular section by Glauert (reference 2) with the latter result valid for small spans. The author (reference 3) then added the general theory for rectangular sections. Of particular interest was the interference in an open rectangular section.

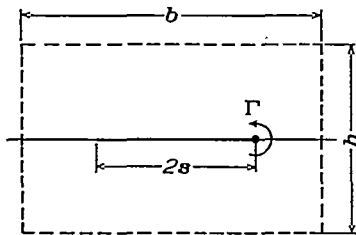


FIGURE 1.

These results were also restricted to small spans. Some time ago Terazawa (reference 4) and subsequently Rosenhead (reference 5) solved the problem of the interference on an airfoil of finite span in a closed rectangular tunnel. It remained for Glauert to bring the result into a form more suitable for calculation; the results are given in reference 6. A very interesting paper by Sanuki and Tani (reference 7) then produced the interference for the elliptic tunnels both open and closed and for airfoils of any span. In the meantime Glauert (reference 8) had already solved the particular case of small spans. It thus appears that the only case not available is the interference on an airfoil of finite span in an open rectangular tunnel. This problem will be studied in the following.

### THE WALL INTERFERENCE ON AN AIRFOIL OF FINITE SPAN IN AN OPEN RECTANGULAR TUNNEL

In figure 1,  $\Gamma$  represents one of the semi-infinite trailing vortices of an airfoil. The airfoil is located

symmetrically in the tunnel which is of the open, or free-jet, type.

Mathematically the problem is now to find a stream function, regular in the interior of the rectangle, and satisfying the boundary condition of zero tangential velocity. The problem leads to elliptic functions due to the double periodicity. It is fortunately possible to obtain the result directly by locating a number of singular points in the exterior region. The arrangement of

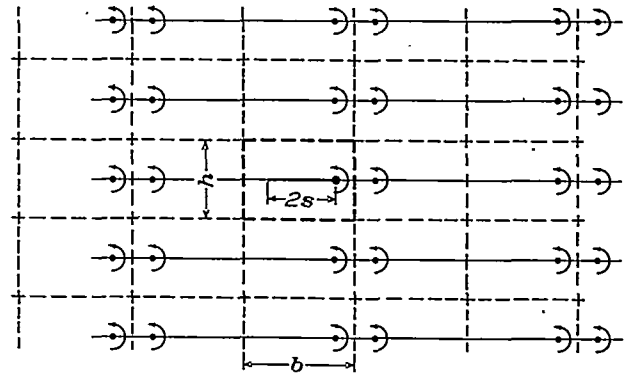


FIGURE 2.

these singular points is shown in figure 2. The singular points are obviously vortices of strength  $\Gamma$ . The tangential velocity is seen to be zero along all bound-

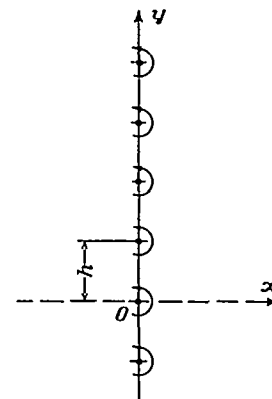


FIGURE 3.

aries of the section by virtue of the existing complete symmetry with respect to each and all boundaries.

We can therefore proceed to consider the stream function due to the external vortex filaments. This function is known for a single infinite row of equidistant vortices. (See reference 9, p. 207.) We shall put down the velocity function for an infinite vertical row

of semi-infinite vortex filaments at  $x=0$  and with a spacing of  $h$  (fig. 3). This is

$$v = \frac{\Gamma}{4h} \frac{\sinh \frac{2\pi x}{h}}{\cosh \frac{2\pi x}{h} - \cos \frac{2\pi y}{h}}$$

where  $v$  is the vertical velocity.

We simplify the result by putting the  $x$  axis through the vortex representing the airfoil. This permits us to put  $y=0$  in the formula, which becomes

$$v = \frac{\Gamma}{4h} \frac{\sinh \frac{2\pi x}{h}}{\cosh \frac{2\pi x}{h} - 1} \text{ or } \frac{\Gamma}{4h} \coth \frac{\pi x}{h}$$

Following Terazawa, we shall now find the total downflow due to this row of vortex filaments by integrating along the  $x$  axis. Thus

$$D = \int v dx = \int \frac{\Gamma}{4h} \coth \frac{\pi x}{h} dx = \frac{\Gamma}{4\pi} \log \sinh \frac{\pi x}{h}$$

We shall now extend the result to include *two* vertical rows as in figure 4. Both rows extend from minus to

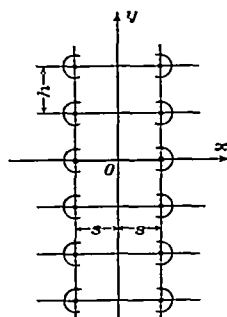


FIGURE 4.

plus infinity; the second row contains filaments of negative sign. The distance between the rows is  $2s$ . Let the original row be located at  $+s$  and the negative one at  $-s$ , respectively, so that the middle point is located at the origin. The stream function now becomes

$$\begin{aligned} D &= \frac{\Gamma}{4\pi} \log \sinh \pi \frac{(x-s)}{h} - \frac{\Gamma}{4\pi} \log \sinh \pi \frac{(x+s)}{h} \\ &= -\frac{\Gamma}{4\pi} \log \frac{\sinh \pi \frac{(x+s)}{h}}{\sinh \pi \frac{(x-s)}{h}} \end{aligned} \quad (1)$$

We may now complete figure 1 by adding the negative vortex filaments as in figure 5. The boundary condition is, of course, still satisfied in proper manner. Notice that we have a number of double rows like the one just considered. We expect to calculate the total downflow between  $-s$  and  $+s$  at the location of the airfoil,  $y=0$ . Because of the right-left symmetry, it is only necessary to determine the downflow between

$x=-s$  and  $x=+s$  due to all vortex rows located on one side of the  $y$  axis. Now, however, the contribution to this downflow due to a certain double row, say the  $n$ th, is numerically the same as the downflow at the location of the  $n$ th row caused by the double row at the origin. It is therefore only necessary to calculate the downflow of this double row located at the origin between the limits  $x=nb-s$  and  $x=nb+s$ .

To obtain the actual downflow due to the first exterior double row at  $x=b$ , we simply put in the

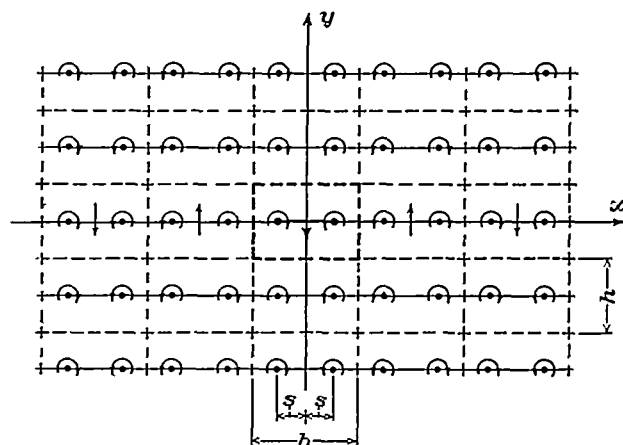


FIGURE 5.

limits  $x=b-s$  and  $x=b+s$  in our equation (1). This gives as the downflow due to this row

$$D = \frac{\Gamma}{4\pi} \left( \log \frac{\sinh \pi \frac{b+2s}{h}}{\sinh \pi \frac{b}{h}} - \log \frac{\sinh \pi \frac{b}{h}}{\sinh \pi \frac{b-2s}{h}} \right)$$

$$D = \frac{\Gamma}{4\pi} \log \frac{\sinh \pi \frac{b+2s}{h} \sinh \pi \frac{b-2s}{h}}{\sinh^2 \pi \frac{b}{h}}$$

$$D = \frac{\Gamma}{4\pi} \log \left( 1 - \frac{\sinh^2 \pi \frac{2s}{h}}{\sinh^2 \pi \frac{b}{h}} \right)$$

For the  $n$ th double row consequently

$$D = \frac{\Gamma}{4\pi} (-1)^n \log \left( 1 - \frac{\sinh^2 \pi \frac{2s}{h}}{\sinh^2 \pi \frac{nb}{h}} \right)$$

and for the entire downflow due to all rows from  $x=-\infty$  to  $x=+\infty$ , except the one at the origin,

$$D = -\frac{\Gamma}{2\pi} \sum_{n=1}^{\infty} (-1)^n \log \left( 1 - \frac{\sinh^2 \pi \frac{2s}{h}}{\sinh^2 \pi \frac{nb}{h}} \right) \quad (2)$$

The double row located at the origin requires a separate treatment. We desire to find the effect of the exterior vortices only; we must therefore eliminate the effect of the vortex pair representing the airfoil at  $y=0$ .

The vertical-flow velocity of one semi-infinite vortex at the origin is  $\frac{\Gamma}{4\pi} \frac{1}{x}$ . Integrated, this gives the downflow

$$D = \frac{\Gamma}{4\pi} \log x$$

Putting a positive vortex at  $-s$  and a negative vortex at  $+s$  leads to a total downflow

$$D = \frac{\Gamma}{4\pi} \log \frac{x+s}{x-s}$$

By adding this expression to (1) we obtain the desired result

$$D_1 = -\frac{\Gamma}{4\pi} \log \frac{\frac{\sinh \pi \frac{x+s}{h}}{\frac{x+s}{h}}}{\frac{\sinh \pi \frac{x-s}{h}}{\frac{x-s}{h}}}$$

We may now put in the limits  $x=-s$  to  $x=+s$ , which gives the downflow of the zero double row

$$D_1 = -\frac{\Gamma}{2\pi} \log \frac{\sinh \pi \frac{2s}{h}}{\frac{2s}{h}}$$

By adding the equations (2) and (3) we obtain finally:

$$D = -\frac{\Gamma}{2\pi} \left( \log \frac{\sinh \pi \frac{2s}{h}}{\frac{2s}{h}} + \sum_{n=1}^{\infty} (-1)^n \log \left( 1 - \frac{\sinh^2 \pi \frac{2s}{h}}{\sinh^2 \pi \frac{nb}{h}} \right) \right) = \frac{\Gamma}{2\pi} \psi$$

where  $\psi$  represents the negative of the quantity in the brackets.

By means of the relation

$$\Gamma 2s \rho V = \frac{1}{2} C_{L\rho} V^2 S$$

where  $V$  is the velocity,  $\rho$  the density of the medium, and  $S$  the area of the airfoil, we obtain

$$\Gamma = \frac{C_L V S}{4s}$$

The angle of deflection of the air stream is given by

$$\epsilon = \frac{v}{V} = \frac{D}{2sV} = \frac{C_L S}{16s^2 \pi} \psi$$

or, with  $bh=C$ , the cross-sectional area of the jet, and  $\frac{b}{h}=r$ , the ratio of tunnel width to tunnel height,

$$\epsilon = \frac{C_L S}{C} \frac{\psi}{4\pi \left(\frac{2s}{b}\right)^2 r} = \frac{C_L S}{C} \delta$$

where  $\delta$  is the wall correction factor as conventionally defined.

Introducing  $\sigma = \frac{2s}{b}$ , the ratio of the span of the airfoil

to the width of the tunnel, we obtain the final result

$$\delta = -\frac{1}{4\pi r \sigma^2} \left\{ \log \frac{\sinh \pi r \sigma}{\pi r \sigma} + \sum_{n=1}^{\infty} (-1)^n \log \left( 1 - \frac{\sinh^2 \pi r \sigma}{\sinh^2 \pi n r} \right) \right\} \quad (4)$$

Table I and figure 6 give the numerical results of the boundary correction factor  $\delta$  for various spans  $\sigma$  for the height-width ratios of practical importance. The series converges so rapidly that the third term already is negligible. Notice that the correction in the

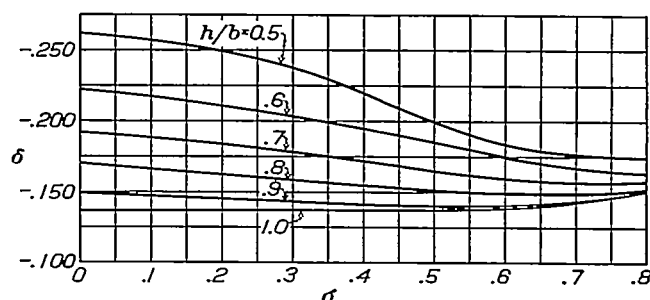


FIGURE 6.—Boundary correction factor  $\delta$  against the span ratio  $\sigma$  for open rectangular wind tunnels of various height-width ratios,  $h/b$ .

square tunnel remains practically constant while the 2:1 tunnel shows a considerable change with the span. The results are strictly true for a constant span loading; however, the vortices  $\Gamma$  may be considered to represent the centers of the trailing tip-vortex systems and the results thus be extended to include any normal span loading with an accuracy sufficient for all practical purposes.

It is interesting to determine the above expression for  $s \rightarrow 0$ .

It appears in the form

$$\delta = \frac{\pi r}{4} \left( -\frac{1}{6} + \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sinh^2 \pi n b/h} \right)$$

or exactly as given for this case by the formula (VIII) on page 6 and by case II on page 8 of a previous report (reference 3) by the author.

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TABLE I.—CORRECTION FACTOR  $\delta$  IN OPEN-THROAT RECTANGULAR WIND TUNNELS

$\lambda = \frac{h}{b}$	$\sigma = \frac{2s}{b}$	$\delta$
1.0	0	0.137
	.2	.137
	.4	.1375
	.6	.139
	.8	.151
.9	0	0.149
	.2	.145
	.4	.142
	.6	.141
	.8	.1515
.8	0	0.171
	.2	.163
	.4	.156
	.6	.149
	.8	.152
.7	0	0.192
	.2	.183
	.4	.172
	.6	.160
	.8	.158
.6	0	0.222
	.2	.211
	.4	.194
	.6	.175
	.8	.164
.5	0	0.262
	.2	.249
	.4	.222
	.6	.183
	.8	.1745